Complex Numbers

$$a+bi \Leftrightarrow r\angle\theta$$

TUTORIAL 3

1. Conversion (Rectangular to Polar form)
2. Conversion (Polar to Rectangular form)
3. Illustrative Examples

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The 21st Century

Representations of ℂ-numbers

$$z \quad \underset{\textit{rectangular}}{= a + bi} \quad \underset{\textit{polar}}{= r\angle\theta} \quad \underset{\textit{trigonometric}}{= r(\cos\theta + i\sin\theta)} \quad \underset{\textit{exponential}}{= re^{i\theta}}$$

The 4 common forms of complex numbers are:

- **Rectangular**
- **Polar**
- **Trigonometric**
- **Exponential**

} Requires a and b

} Requires r and θ

Only the exponential form must have the angle in rads.!

Very Important Formulations!

Conversion of ℂ-numbers

(From one form to another)

$$\underset{}{z} \quad \underset{\textit{rectangular}}{= a + bi} \quad \underset{\textit{polar}}{= r\angle\theta} \quad \underset{\textit{trigonometric}}{= r(\cos\theta + i\sin\theta)} \quad \underset{\textit{exponential}}{= re^{i\theta}}$$

Decompositions	**Formulations** $z \in \mathbb{C}, \quad a,b,r \in \mathbb{R}$
Modulus	$\operatorname{mod} z = \lvert z \rvert = r = \sqrt{a^2 + b^2}$
Argument	$\arg z = \theta = \tan^{-1}(b/a)$
Real Part	$\operatorname{Re} z = a = \lvert z \rvert \cos\theta = r\cos\theta$
Imaginary Part	$\operatorname{Im} z = b = \lvert z \rvert \sin\theta = r\sin\theta$

Complex Numbers Conversion, $a+bi \rightarrow r\angle\theta$

Example 3. Convert the following complex numbers that are in *Rectangular form* to the *Polar form*.

$$(a)\qquad z = 5+12i$$

$$(b)\qquad z = -2+2i$$

$$(c)\qquad z = -5-8i$$

$$(d)\qquad z = 3-4i$$

Express all angles in radians, unless otherwise stated, such that.

$$-\pi < \theta \leq \pi$$

Complex Numbers
Conversion, $a+bi \to r\angle\theta$

Response 3(a). Now, $z = 5 + 12i$ is in Quadrant I.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \operatorname{mod} z \\ &= |5+12i| \\ &= \sqrt{5^2+12^2} \\ &= 13 \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(5+12i) \\ &= \tan^{-1}\left(\frac{12}{5}\right) \\ &= 1.176^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z|\angle\theta \\ &= 13\angle 1.176^c \end{aligned}$$

Note that $r = |z|$

Complex Numbers
Conversion, $a+bi \to r\angle\theta$

Response 3(b). Now, $z=-2+2i$ is in Quadrant II.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned}|z| &= \operatorname{mod} z\\ &= |-2+2i|\\ &= \sqrt{2^2+2^2}\\ &= \sqrt{4\times 2}\\ &= 2\sqrt{2}\ \text{units}\end{aligned}$$

$$\begin{aligned}\theta &= \arg z\\ &= \arg(-2+2i)\\ &= \pi+\tan^{-1}\left(\frac{2}{-2}\right)\\ &= \frac{3\pi}{4}\end{aligned}$$

The solution is:

$$\begin{aligned}z &= |z|\angle\theta\\ &= 2\sqrt{2}\angle\frac{3\pi}{4}\end{aligned}$$

Complex Numbers
Conversion, $a+bi \rightarrow r\angle\theta$

Response 3(c). Now, $z = -5 - 8i$ is in Quadrant III.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$
\begin{aligned}
|z| &= \operatorname{mod} z \\
&= |-5-8i| \\
&= \sqrt{5^2+8^2} \\
&= \sqrt{25+64} \\
&= \sqrt{89} \text{ units}
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \arg z \\
&= \arg(-5-8i) \\
&= -\pi + \tan^{-1}\left(\frac{-8}{-5}\right) \\
&= -2.129^c
\end{aligned}
$$

The solution is:

$$
\begin{aligned}
z &= |z|\angle\theta \\
&= \sqrt{89}\angle -2.129^c
\end{aligned}
$$

Complex Numbers
Conversion, $a+bi \to r\angle\theta$

Response 3(d). Now, $z = 3 - 4i$ is in Quadrant IV.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$
\begin{aligned}
|z| &= \mathrm{mod}\, z \\
&= |z = 3 - 4i| \\
&= \sqrt{3^2 + 4^2} \\
&= 5 \text{ units}
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \arg z \\
&= \arg(3 - 4i) \\
&= \tan^{-1}\left(\frac{-4}{3}\right) \\
&= -0.927^c
\end{aligned}
$$

The solution is:

$$
\begin{aligned}
z &= |z|\angle\theta \\
&= 5\angle - 0.927^c
\end{aligned}
$$

Complex Numbers Conversion, $a+bi \rightarrow r\angle\theta$

Question 4. Convert the following complex numbers that are in *Rectangular form* to the *Polar form*.

$$(a) \qquad z = 7 + 24i$$

$$(b) \qquad z = -1 + \sqrt{3}i$$

$$(c) \qquad z = -1 - 3i$$

$$(d) \qquad z = 6 - 8i$$

Express all angles in radians, unless otherwise stated, such that.

$$-\pi < \theta \leq \pi$$

Complex Numbers
Conversion, $r\angle\theta \rightarrow a+bi$

Example 4 Convert the following complex numbers that are in *Polar form* to the *Rectangular form*.

$$(a) \qquad z = 100\angle 60^\circ$$

$$(b) \qquad z = 42\angle 135^\circ$$

$$(c) \qquad z = 7\angle -2.150^c$$

$$(d) \qquad z = 9\angle -0.152^c$$

All angles in radians, unless otherwise stated.

Complex Numbers
Conversion, $r\angle\theta \rightarrow a{+}bi$

Response 4(a). Recall: $z = 100\angle 60^\circ$

First, *compute the real and imaginary parts.* Then state the final solution in *rectangular form.*

$$\begin{aligned} a &= \operatorname{Re} z \\ &= r\cos\theta \\ &= 100\cos 60^\circ \\ &= 100\times 0.5 \\ &= 50.0 \end{aligned}$$

$$\begin{aligned} b &= \operatorname{Im} z \\ &= r\sin\theta \\ &= 100\sin 60^\circ \\ &= 100\times 0.8660 \\ &= 86.6 \end{aligned}$$

Work to 4 sig. figs. and round-off to 3 sig. figs.

The solution is:

$$\begin{aligned} z &= a + bi \\ &= 50.0 + 86.6i \end{aligned}$$

Complex Numbers
Conversion, $r\angle\theta \rightarrow a+bi$

Response 4(b). Recall: $z = 42\angle 135^\circ$

Again, *compute the real and imaginary parts*, and then state the final solution in *rectangular form*.

$$\begin{aligned} a &= \operatorname{Re} z \\ &= r\cos\theta \\ &= 42\cos 135^\circ \\ &= -21\sqrt{2} \end{aligned}$$

$$\begin{aligned} b &= \operatorname{Im} z \\ &= r\sin\theta \\ &= 42\sin 135^\circ \\ &= 21\sqrt{2} \end{aligned}$$

The solution is:

$$\begin{aligned} z &= a + bi \\ &= -21\sqrt{2} + 21\sqrt{2}i \end{aligned}$$

Complex Numbers
Conversion, $r\angle\theta \to a+bi$

Response 4(c). Recall: $z = 7\angle -2.150^c$

A more systematic approach is to go from *polar* to '*trigonometric*' and then to the *rectangular form*. So,

$$
\begin{aligned}
z &= 7\angle -2.150^c \\
&= r\left(\cos\theta + i\sin\theta\right) && \text{<--- skippable step} \\
&= 7\left[\cos\left(-2.150^c\right) + i\sin\left(-2.150^c\right)\right] \\
&= 7\left(-0.5474 - 0.5474i\right) && \text{<--- skippable step} \\
&= -3.831 - 5.858i
\end{aligned}
$$

Complex Numbers
Conversion, $r\angle\theta \rightarrow a+bi$

Response 4(d). Recall: $z = 9\angle -0.152^{c}$

A most efficient approach is to go from *polar* to '*trigonometric*' and then to the *rectangular form*. So,

$$\begin{aligned} z &= 9\angle -0.152^{c} \\ &= 9\left[\cos\left(-0.152^{c}\right)+i\sin\left(-0.152^{c}\right)\right] \\ &= 8.90-1.38i \end{aligned}$$

Square brackets must be expanded with care

Complex Numbers Conversion, $r\angle\theta \to a+bi$

Question 5. Convert the following complex numbers that are in *Polar form* to the *Rectangular form*.

$$(a) \qquad z = 10\angle 30^{\circ}$$

$$(b) \qquad z = 8\angle 120^{\circ}$$

$$(c) \qquad z = 3\angle -2.50^{c}$$

$$(d) \qquad z = 5\angle -1^{c}$$

All angles in radians, unless otherwise stated.

Tutorial 3

Complex Numbers

Complex Numbers Conversion, $a+bi \rightarrow r\angle\theta$

Question 4. Convert the following complex numbers that are in *Rectangular form* to the *Polar form*.

$$(a) \qquad z = 7+24i$$

$$(b) \qquad z = -1+\sqrt{3}i$$

$$(c) \qquad z = -1-3i$$

$$(d) \qquad z = 6-8i$$

Express all angles in radians, unless otherwise stated, such that.

$$-\pi < \theta \leq \pi$$

Tutorial Solutions
Complex Numbers

Solution 4(a). Now, $z = 7 + 24i$ is in Quadrant I.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \operatorname{mod} z \\ &= |7 + 24i| \\ &= \sqrt{7^2 + 24^2} \\ &= 25 \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(7 + 24i) \\ &= \tan^{-1}\left(\frac{24}{7}\right) \\ &= 1.287^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= 25 \angle 1.287^c \end{aligned}$$

Tutorial Solutions
Complex Numbers

Solution 4(b). Now, $z = -1 + \sqrt{3}i$ is in Quadrant II.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \operatorname{mod} z \\ &= \left|-1+\sqrt{3}i\right| \\ &= \sqrt{1^2 + \left(\sqrt{3}\right)^2} \\ &= 2 \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg\left(-1+\sqrt{3}i\right) \\ &= \pi + \tan^{-1}\left(-\sqrt{3}\right) \\ &= \frac{2\pi}{3} \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= 2\angle \frac{2\pi}{3} \end{aligned}$$

Tutorial Solutions
Complex Numbers

Solution 4(c). Now, $z = -1 - 3i$ is in Quadrant III.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$
\begin{aligned}
|z| &= \operatorname{mod} z \\
&= |-1 - 3i\ | \\
&= \sqrt{1^2 + 3^2} \\
&= \sqrt{10}\ \text{ units}
\end{aligned}
$$

$$
\begin{aligned}
\theta &= \arg z \\
&= \arg(-1 - 3i\ ) \\
&= -\pi + \tan^{-1}\left(\frac{-3}{-1}\right) \\
&= -1.893^c
\end{aligned}
$$

The solution is:

$$
\begin{aligned}
z &= |z| \angle \theta \\
&= \sqrt{10} \angle -1.893^c
\end{aligned}
$$

Tutorial Solutions
Complex Numbers

Solution 4(d). Now, $z = 6 - 8i$ is in Quadrant IV.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \operatorname{mod} z \\ &= |z = 6 - 8i| \\ &= \sqrt{6^2 + 8^2} \\ &= 10 \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(6 - 8i) \\ &= \tan^{-1}\left(\frac{-8}{6}\right) \\ &= -0.927^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= 10 \angle -0.927^c \end{aligned}$$

Complex Numbers Conversion, $r\angle\theta \rightarrow a+bi$

Question 5. Convert the following complex numbers that are in *Polar form* to the *Rectangular form*.

$(a) \qquad z = 10\angle 30^{\circ}$

$(b) \qquad z = 8\angle 120^{\circ}$

$(c) \qquad z = 3\angle -2.50^{c}$

$(d) \qquad z = 5\angle -1^{c}$

All angles in radians, unless otherwise stated.

Tutorial Solutions
Complex Numbers

Solution 5(a). Recall: $z = 10\angle 30^\circ$

First, *compute the real and imaginary parts.* Then state the final solution in *rectangular form*.

$$\begin{aligned} a &= \operatorname{Re} z \\ &= r\cos\theta \\ &= 10\cos 30^\circ \\ &= 10 \times 0.8660 \\ &= 8.66 \end{aligned}$$

$$\begin{aligned} b &= \operatorname{Im} z \\ &= r\sin\theta \\ &= 10\sin 30^\circ \\ &= 10 \times 0.5 \\ &= 5.00 \end{aligned}$$

The solution is:

$$\begin{aligned} z &= a + bi \\ &= 8.66 + 5.00i \end{aligned}$$

Work to 4 sig. figs. and round-off to 3 sig. figs.

Tutorial Solutions
Complex Numbers

Solution 5(b). Recall: $z = 8\angle 120^\circ$

Again, *compute the real and imaginary parts*, and then state the final solution in *rectangular form*.

$$\begin{aligned} a &= \operatorname{Re} z \\ &= r\cos\theta \\ &= 8\cos 120^\circ \\ &= 8 \times \frac{-1}{2} \\ &= -4 \end{aligned}$$

$$\begin{aligned} b &= \operatorname{Im} z \\ &= r\sin\theta \\ &= 8\sin 120^\circ \\ &= 8 \times \frac{\sqrt{3}}{2} \\ &= 4\sqrt{3} \end{aligned}$$

The solution is:

$$\begin{aligned} z &= a + bi \\ &= -4 + 4\sqrt{3}i \end{aligned}$$

Note that the final solution is EXACT.

Tutorial Solutions
Complex Numbers

Solution 5(c). Recall: $z = 3\angle -2.50^c$

A more systematic approach is to go from *polar* to '*trigonometric*' and then to the *rectangular form*. So,

$$\begin{aligned} z &= 3\angle -2.50^c \\ &= r(\cos\theta + i\sin\theta) && \text{<--- skippable step} \\ &= 3\left[\cos\left(-2.50^c\right) + i\sin\left(-2.50^c\right)\right] \\ &= 3(-0.8011 - 0.5985i) && \text{<--- skippable step} \\ &= -2.40 - 1.80i \end{aligned}$$

Tutorial Solutions
Complex Numbers

Solution 5(d). Recall: $z = 5\angle -1^c$

A most efficient approach is to go from *polar* to '*trigonometric*' and then to the *rectangular form*. So,

$$
\begin{aligned}
z &= 5\angle -1^c \\
&= 5\left[\cos\left(-1^c\right) + i\sin\left(-1^c\right)\right] \\
&= 2.70 - 4.21i
\end{aligned}
$$

Square brackets must be expanded with care

Complex Numbers

Thank You

For more information, please visit:
http://www.boszik.net/mse