

# Complex Numbers

$$a + bi \iff r \angle \theta$$

## TUTORIAL 3

1. **Conversion** (Rectangular to Polar form)
2. **Conversion** (Polar to Rectangular form)
3. Illustrative **Examples**

G. David Boswell | Chronicles of BÖŞZİK Inc.™

**The 21st Century**

# Representations of $\mathbb{C}$ -numbers

$$z = a + bi \quad = r \angle \theta \quad = r(\cos \theta + i \sin \theta) \quad = re^{i\theta}$$

*rectangular*                      *polar*                      *trigonometric*                      *exponential*

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The 4 common forms of complex numbers are:

- **Rectangular**

- **Polar**

- **Trigonometric**

- **Exponential**

} Requires  $a$  and  $b$

} Requires  $r$  and  $\theta$

Only the exponential form must have the angle in rads.!

Very Important  
Formulations!

# Conversion of $\mathbb{C}$ -numbers

(From one form to another)

$$z = a + bi \quad = r \angle \theta \quad = r(\cos \theta + i \sin \theta) \quad = re^{i\theta}$$

*rectangular*                      *polar*                      *trigonometric*                      *exponential*

**Decompositions**

**Formulations**

$$z \in \mathbb{C}, \quad a, b, r \in \mathbb{R}$$

**Modulus**

$$\text{mod } z = |z| = r = \sqrt{a^2 + b^2}$$

**Argument**

$$\text{arg } z = \theta = \tan^{-1}(b/a)$$

**Real Part**

$$\text{Re } z = a = |z| \cos \theta = r \cos \theta$$

**Imaginary Part**

$$\text{Im } z = b = |z| \sin \theta = r \sin \theta$$

# Complex Numbers

## Conversion, $a+bi \rightarrow r\angle\theta$

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**Example 3.** Convert the following complex numbers that are in *Rectangular form* to the *Polar form*.

$$(a) \quad z = 5 + 12i$$

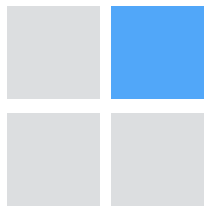
$$(b) \quad z = -2 + 2i$$

$$(c) \quad z = -5 - 8i$$

$$(d) \quad z = 3 - 4i$$

Express all angles in radians, unless otherwise stated, such that.

$$-\pi < \theta \leq \pi$$



# Complex Numbers

## Conversion, $a+bi \rightarrow r \angle \theta$

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**Response 3(a).** Now,  $z = 5 + 12i$  is in **Quadrant I**.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

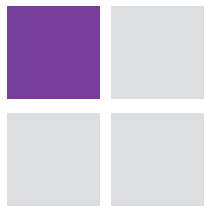
$$\begin{aligned} |z| &= \text{mod } z \\ &= |5 + 12i| \\ &= \sqrt{5^2 + 12^2} \\ &= 13 \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(5 + 12i) \\ &= \tan^{-1}\left(\frac{12}{5}\right) \\ &= 1.176^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= 13 \angle 1.176^c \end{aligned}$$

Note that  $r = |z|$



# Complex Numbers

## Conversion, $a+bi \rightarrow r \angle \theta$

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**Response 3(b).** Now,  $z = -2 + 2i$  is in **Quadrant II**.

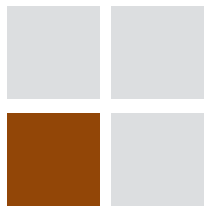
First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \text{mod } z \\ &= |-2 + 2i| \\ &= \sqrt{2^2 + 2^2} \\ &= \sqrt{4 \times 2} \\ &= 2\sqrt{2} \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(-2 + 2i) \\ &= \pi + \tan^{-1}\left(\frac{2}{-2}\right) \\ &= \frac{3\pi}{4} \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= 2\sqrt{2} \angle \frac{3\pi}{4} \end{aligned}$$



# Complex Numbers

## Conversion, $a+bi \rightarrow r \angle \theta$

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**Response 3(c).** Now,  $z = -5 - 8i$  is in **Quadrant III**.

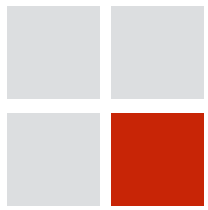
First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \text{mod } z \\ &= |-5 - 8i| \\ &= \sqrt{5^2 + 8^2} \\ &= \sqrt{25 + 64} \\ &= \sqrt{89} \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(-5 - 8i) \\ &= -\pi + \tan^{-1}\left(\frac{-8}{-5}\right) \\ &= -2.129^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= \sqrt{89} \angle -2.129^c \end{aligned}$$



# Complex Numbers

## Conversion, $a+bi \rightarrow r \angle \theta$

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**Response 3(d).** Now,  $z = 3 - 4i$  is in **Quadrant IV**.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \text{mod } z \\ &= |z = 3 - 4i| \\ &= \sqrt{3^2 + 4^2} \\ &= 5 \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(3 - 4i) \\ &= \tan^{-1}\left(\frac{-4}{3}\right) \\ &= -0.927^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= 5 \angle -0.927^c \end{aligned}$$





# Complex Numbers

## Conversion, $a+bi \rightarrow r \angle \theta$

**Question 4.** Convert the following complex numbers that are in *Rectangular form* to the *Polar form*.

(a)  $z = 7 + 24i$

(b)  $z = -1 + \sqrt{3}i$

(c)  $z = -1 - 3i$

(d)  $z = 6 - 8i$

Express all angles in radians, unless otherwise stated, such that.

$$-\pi < \theta \leq \pi$$

# Complex Numbers

## Conversion, $r\angle\theta \rightarrow a+bi$

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**Example 4** Convert the following complex numbers that are in *Polar form* to the *Rectangular form*.

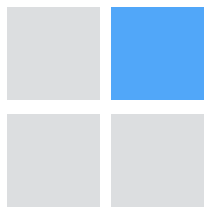
$$(a) \quad z = 100\angle 60^\circ$$

$$(b) \quad z = 42\angle 135^\circ$$

$$(c) \quad z = 7\angle -2.150^c$$

$$(d) \quad z = 9\angle -0.152^c$$

All angles in radians, unless otherwise stated.



# Complex Numbers

## Conversion, $r\angle\theta \rightarrow a+bi$

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**Response 4(a).** Recall:  $z = 100\angle 60^\circ$

First, *compute the real and imaginary parts*. Then state the final solution in *rectangular form*.

$$a = \operatorname{Re} z$$

$$= r \cos \theta$$

$$= 100 \cos 60^\circ$$

$$= 100 \times 0.5$$

$$= 50.0$$

$$b = \operatorname{Im} z$$

$$= r \sin \theta$$

$$= 100 \sin 60^\circ$$

$$= 100 \times 0.8660$$

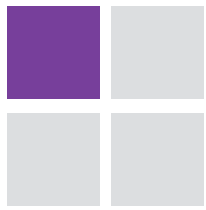
$$= 86.6$$

The solution is:

$$z = a + bi$$

$$= 50.0 + 86.6i$$

Work to 4 sig. figs. and round-off to 3 sig. figs.



# Complex Numbers

## Conversion, $r\angle\theta \rightarrow a+bi$

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**Response 4(b).** Recall:  $z = 42\angle 135^\circ$

Again, compute the real and imaginary parts, and then state the final solution in rectangular form.

$$a = \operatorname{Re} z$$

$$= r \cos \theta$$

$$= 42 \cos 135^\circ$$

$$= -21\sqrt{2}$$

$$b = \operatorname{Im} z$$

$$= r \sin \theta$$

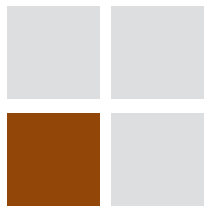
$$= 42 \sin 135^\circ$$

$$= 21\sqrt{2}$$

The solution is:

$$z = a + bi$$

$$= -21\sqrt{2} + 21\sqrt{2}i$$



# Complex Numbers

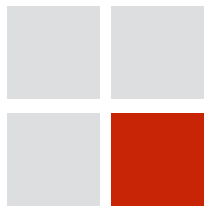
## Conversion, $r \angle \theta \rightarrow a+bi$

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**Response 4(c).** Recall:  $z = 7 \angle -2.150^\circ$

A **more systematic approach** is to go from *polar* to ‘*trigonometric*’ and then to the *rectangular form*. So,

$$\begin{aligned} z &= 7 \angle -2.150^\circ \\ &= r(\cos \theta + i \sin \theta) && \text{<--- skippable step} \\ &= 7 \left[ \cos(-2.150^\circ) + i \sin(-2.150^\circ) \right] \\ &= 7(-0.5474 - 0.5474i) && \text{<--- skippable step} \\ &= -3.831 - 5.858i \end{aligned}$$



# Complex Numbers

## Conversion, $r \angle \theta \rightarrow a+bi$

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**Response 4(d).** Recall:  $z = 9 \angle -0.152^\circ$

A **most efficient approach** is to go from *polar* to '*trigonometric*' and then to the *rectangular form*. So,

$$\begin{aligned} z &= 9 \angle -0.152^\circ \\ &= 9 \left[ \cos(-0.152^\circ) + i \sin(-0.152^\circ) \right] \\ &= 8.90 - 1.38i \end{aligned}$$

Square brackets must be expanded with care



# Complex Numbers

## Conversion, $r\angle\theta \rightarrow a+bi$

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**Question 5.** Convert the following complex numbers that are in *Polar form* to the *Rectangular form*.

(a)  $z = 10\angle 30^\circ$

(b)  $z = 8\angle 120^\circ$

(c)  $z = 3\angle -2.50^c$

(d)  $z = 5\angle -1^c$

All angles in radians, unless otherwise stated.

# Tutorial 3

## Complex Numbers

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# Complex Numbers

## Conversion, $a+bi \rightarrow r \angle \theta$

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**Question 4.** Convert the following complex numbers that are in *Rectangular form* to the *Polar form*.

(a)  $z = 7 + 24i$

(b)  $z = -1 + \sqrt{3}i$

(c)  $z = -1 - 3i$

(d)  $z = 6 - 8i$

Express all angles in radians, unless otherwise stated, such that.

$$-\pi < \theta \leq \pi$$



# Tutorial Solutions

## Complex Numbers

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**Solution 4(a).** Now,  $z = 7 + 24i$  is in **Quadrant I**.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \text{mod } z \\ &= |7 + 24i| \\ &= \sqrt{7^2 + 24^2} \\ &= 25 \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(7 + 24i) \\ &= \tan^{-1}\left(\frac{24}{7}\right) \\ &= 1.287^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= 25 \angle 1.287^c \end{aligned}$$



# Tutorial Solutions

## Complex Numbers

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**Solution 4(b).** Now,  $z = -1 + \sqrt{3}i$  is in **Quadrant II**.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$|z| = \text{mod } z$$

$$= |-1 + \sqrt{3}i|$$

$$= \sqrt{1^2 + (\sqrt{3})^2}$$

$$= 2 \text{ units}$$

$$\theta = \arg z$$

$$= \arg(-1 + \sqrt{3}i)$$

$$= \pi + \tan^{-1}(-\sqrt{3})$$

$$= \frac{2\pi}{3}$$

The solution is:

$$z = |z| \angle \theta$$

$$= 2 \angle \frac{2\pi}{3}$$



# Tutorial Solutions

## Complex Numbers

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**Solution 4(c).** Now,  $z = -1 - 3i$  is in **Quadrant III**.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \text{mod } z \\ &= |-1 - 3i| \\ &= \sqrt{1^2 + 3^2} \\ &= \sqrt{10} \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(-1 - 3i) \\ &= -\pi + \tan^{-1}\left(\frac{-3}{-1}\right) \\ &= -1.893^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= \sqrt{10} \angle -1.893^c \end{aligned}$$



# Tutorial Solutions

## Complex Numbers

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**Solution 4(d).** Now,  $z = 6 - 8i$  is in **Quadrant IV**.

First, *compute the modulus and argument*, then state the final solution in *polar form*.

$$\begin{aligned} |z| &= \text{mod } z \\ &= |z = 6 - 8i| \\ &= \sqrt{6^2 + 8^2} \\ &= 10 \text{ units} \end{aligned}$$

$$\begin{aligned} \theta &= \arg z \\ &= \arg(6 - 8i) \\ &= \tan^{-1}\left(\frac{-8}{6}\right) \\ &= -0.927^c \end{aligned}$$

The solution is:

$$\begin{aligned} z &= |z| \angle \theta \\ &= 10 \angle -0.927^c \end{aligned}$$



# Complex Numbers

## Conversion, $r\angle\theta \rightarrow a+bi$

---

**Question 5.** Convert the following complex numbers that are in *Polar form* to the *Rectangular form*.

(a)  $z = 10\angle 30^\circ$

(b)  $z = 8\angle 120^\circ$

(c)  $z = 3\angle -2.50^c$

(d)  $z = 5\angle -1^c$

All angles in radians, unless otherwise stated.



# Tutorial Solutions

## Complex Numbers

**Solution 5(a).** Recall:  $z = 10 \angle 30^\circ$

First, *compute the real and imaginary parts*. Then state the final solution in *rectangular form*.

$$\begin{aligned} a &= \operatorname{Re} z \\ &= r \cos \theta \\ &= 10 \cos 30^\circ \\ &= 10 \times 0.8660 \\ &= 8.66 \end{aligned}$$

$$\begin{aligned} b &= \operatorname{Im} z \\ &= r \sin \theta \\ &= 10 \sin 30^\circ \\ &= 10 \times 0.5 \\ &= 5.00 \end{aligned}$$

The solution is:

$$\begin{aligned} z &= a + bi \\ &= 8.66 + 5.00i \end{aligned}$$

Work to 4 sig. figs. and round-off to 3 sig. figs.



# Tutorial Solutions

## Complex Numbers

**Solution 5(b).** Recall:  $z = 8 \angle 120^\circ$

Again, compute the real and imaginary parts, and then state the final solution in rectangular form.

$$\begin{aligned}a &= \operatorname{Re} z \\ &= r \cos \theta \\ &= 8 \cos 120^\circ \\ &= 8 \times \frac{-1}{2} \\ &= -4\end{aligned}$$

$$\begin{aligned}b &= \operatorname{Im} z \\ &= r \sin \theta \\ &= 8 \sin 120^\circ \\ &= 8 \times \frac{\sqrt{3}}{2} \\ &= 4\sqrt{3}\end{aligned}$$

The solution is:

$$\begin{aligned}z &= a + bi \\ &= -4 + 4\sqrt{3}i\end{aligned}$$

Note that the final solution is EXACT.





# Tutorial Solutions

## Complex Numbers

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**Solution 5(c).** Recall:  $z = 3\angle -2.50^\circ$

A **more systematic approach** is to go from *polar* to ‘*trigonometric*’ and then to the *rectangular form*. So,

$$\begin{aligned}z &= 3\angle -2.50^\circ \\ &= r(\cos\theta + i\sin\theta) && \text{<--- skippable step} \\ &= 3\left[\cos(-2.50^\circ) + i\sin(-2.50^\circ)\right] \\ &= 3(-0.8011 - 0.5985i) && \text{<--- skippable step} \\ &= -2.40 - 1.80i\end{aligned}$$



# Tutorial Solutions

## Complex Numbers

**Solution 5(d).** Recall:  $z = 5 \angle -1^\circ$

A **most efficient approach** is to go from *polar* to '*trigonometric*' and then to the *rectangular form*. So,

$$\begin{aligned}z &= 5 \angle -1^\circ \\ &= 5 \left[ \cos(-1^\circ) + i \sin(-1^\circ) \right] \\ &= 2.70 - 4.21i\end{aligned}$$

←  
Square brackets must  
be expanded with care

# Complex Numbers

Thank You

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