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Institution : Hampton School

Subject : Pure Math, Unit 1

Assessment : WS03 - Reasoning and Logic

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1 (b) The quantity a is the **inverse** of b. Furthermore, the *converse* of this statement is also true.

1 (c) (i) Given $x, y \in \mathbb{R}$ and the defined operations $x \odot y = 4x + 5y$, then

$4x \in \mathbb{R}$, Multiplication is closed on $\mathbb{R}$

$5y \in \mathbb{R}$, Multiplication is closed on $\mathbb{R}$

$4x + 5y \in \mathbb{R}$, Addition is closed on $\mathbb{R}$

Then, *the binary operator,* $\odot$*, is closed* on $\mathbb{R}$.

1 (c) (ii) Given $x, y, z \in \mathbb{R}$ and $x \odot y = 4x + 5y$, then

$$\begin{aligned}(x \odot y) \odot z &= 4(x \odot y) + 5z \\ &= 4(4x + 5y) + 5z \\ &= 16x + 20y + 5z\end{aligned}$$

Similarly,

$$\begin{aligned}x \odot (y \odot z) &= 4x + 5(y \odot z) \\ &= 4x + 5(4y + 5z) \\ &= 4x + 20y + 25z \\ &\neq (x \odot y) \odot z\end{aligned}$$

Then, *the binary operator,* $\odot$ *is not associative* on $\mathbb{R}$.

1 (c) (iii) Given $x, y, z \in \mathbb{R}$ and $x \odot y = 4x + 5y$, then

$$\begin{aligned} x \odot (y+z) &= 4x + 5(y+z) \\ &= 4x + 5(y+y) \\ &= 4x + 5y + 5z \end{aligned}$$

Similarly,

$$\begin{aligned} (x \odot y) + (x \odot z) &= (4x+5y) + (4x+5z) \\ &= 8x + 5y + 5z \\ &\neq x \odot (y+z) \end{aligned}$$

Then, *the binary operator,* $\odot$ *is not distributive* on $\mathbb{R}$.

2 (a) Given $x, y \in \mathbb{R}$ and the defined operations $x \otimes y = 2x^2 + 2y^2 - 3$, then

$x^2 \in \mathbb{R}$,	Multiplication is closed on $\mathbb{R}$
$2x^2 \in \mathbb{R}$,	″
$y^2 \in \mathbb{R}$,	″
$2y^2 \in \mathbb{R}$,	″
$2x^2 + 2y^2 \in \mathbb{R}$,	Addition is closed on $\mathbb{R}$
$2x^2 + 2y^2 - 3 \in \mathbb{R}$,	Subtraction is closed on $\mathbb{R}$

Hence, *the binary operator,* $\otimes$ *is* closed on $\mathbb{R}$.

2 (b) Given $x, y \in \mathbb{R}$ and $x \otimes y = 2x^2 + 2y^2 - 3$, then

$$\begin{aligned} y \otimes x &= 2y^2 + 2x^2 - 3 \\ &= 2x^2 + 2y^2 - 3 \qquad \text{since addition is commutative} \\ &\equiv x \otimes y \end{aligned}$$

Then, *the binary operator,* $\otimes$ *is indeed commutative* on $\mathbb{R}$.

2 (c) Given $x, y, z \in \mathbb{R}$ and $x \otimes y = 2x^2 + 2y^2 - 3$, then

$$(x \otimes y) \otimes z = 2(x \otimes y)^2 + 2z^2 - 3$$
$$= 2(2x^2 + 2y^2 - 3)^2 + 2z^2 - 3$$

Similarly,

$$x \otimes (y \otimes z) = 2x^2 + 2(y \odot z)^2 - 3$$
$$= 2x^2 + 2(2y^2 + 2z^2 - 3)^2 - 3$$
$$\neq (x \otimes y) \otimes z$$

Then, *the binary operator,* $\otimes$ *is not associative* on $\mathbb{R}$.

2 (d) Given $x, y, z \in \mathbb{R}$ and $x \otimes y = 2x^2 + 2y^2 - 3$, then

$$x \otimes (y + z) = 2x^2 + 2(y + z)^2 - 3$$
$$= 2x^2 + 2(y^2 + 2yz + z^2) - 3$$
$$= 2x^2 + 2y^2 + 4yz + 2z^2 - 3$$

Similarly,

$$(x \otimes y) + (x \otimes z) = (2x^2 + 2y^2 - 3) + (2x^2 + 2z^2 - 3)$$
$$= 4x^2 + 2y^2 + 2z^2 - 6$$
$$\neq x \otimes (y + z)$$

Then, *the binary operator,* $\otimes$ *is not distributive* on $\mathbb{R}$.

3 (a) Given $x, y \in \mathbb{R}$ and $x \otimes y = 3x + y^2 - 10$, then

$$\begin{aligned} 2 \otimes (-3) &= 3(2) + (-3)^2 - 10 \\ &= 6 + 9 - 10 \\ &= 15 - 10 \\ &= 6 \end{aligned}$$

and

$$\begin{aligned} (-3) \otimes (2) &= 3(-3) + (2)^2 - 10 \\ &= -9 + 4 - 10 \\ &= 4 - 19 \\ &= -15 \end{aligned}$$

Comments

Since $2 \otimes (-3) \neq (-3) \otimes (2)$, then the binary operator, $\otimes$ is not commutative on $\mathbb{R}$.

3 (b) Given $x \otimes y = 3x^2 + y^2 - 10$, then $x \otimes x = 0$ implies

$$\begin{aligned} 3x + x^2 - 10 &= 0 \\ x^2 + 3x - 10 &= 0 \\ x^2 - 5x + 2x - 10 &= 0 \\ x(x - 5) + 2(x - 5) &= 0 \\ (x + 2)(x - 5) &= 0 \end{aligned}$$

Hence, either $x = -2$ or $x = 5$

End of Solutions (EOS)

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