(b) A curve has equation $y = x + \dfrac{6}{x^2}$.

(i) Show that $x\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} = 3$. [5]

(ii) Find the equation of the normal to the curve at the point where $x = 1$. [4]

b(i) Given $y = x + \dfrac{6}{x^2}$

$$= x + 6x^{-2}$$

$$\therefore \quad \frac{dy}{dx} = \frac{d}{dx}\left(x + 6x^{-2}\right)$$

$$= 1 - 12x^{-3}$$

$$= 1 - \frac{12}{x^3}$$

and $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(1 - 12x^{-3}\right)$$

$$= 36x^{-4}$$

$$= \frac{36}{x^4}$$

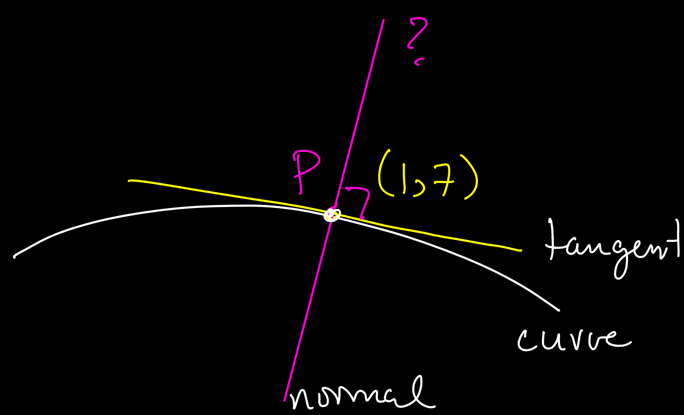
PROOF: LHS $= x y'' + 3y'$

$$= x\left(\frac{36}{x^4}\right) + 3\left(1 - \frac{12}{x^3}\right)$$

$$= \frac{36}{x^3} + 3 - \frac{36}{x^3}$$

$$= 3$$

$$= \text{RHS}$$



b(ii) For the curve,

$$y = x + \frac{6}{x^2}$$

and $$\frac{dy}{dx} = 1 - \frac{12}{x^3}$$

obtain the gradient function early!

at P, $$y(1) = 1 + \frac{6}{1^2}$$

$$= 7$$

Gradient at P is

$$m_1 = \left.\frac{dy}{dx}\right|_{x=1}$$

(tangent concept in calculus)

$$= 1 - \frac{12}{1^3}$$

$$= -11$$

Hence, the gradient of the Normal at P is

$$m_2 \triangleq \frac{1}{m_1}$$

(definition)

$$= \frac{1}{11}$$

So the req. eqn. of the normal is

$$y - y_P = m_2 (x - x_P)$$

$$y - 7 = \frac{1}{11}(x - 1)$$

$$11y - 77 = x - 1$$

$$x - 11y + 76 = 0$$

Question Differential Equations

Given the differential Equation

$$\frac{d^2y}{dx^2} = 3x, \quad \ldots\ldots\ldots (1)$$

find the general solution $y(x)$.

Solution Using integration to reverse the differentiation process, then

Integrating both sides of Eqn(1) gives

$$\frac{dy}{dx} = \int 3x \, dx$$

$$= \frac{3x^2}{2} + C_1$$

Integrating this result again, we get

$$y = \int \frac{3x^2}{2} + C_1 \, dx$$

$$\therefore \quad y(x) = \frac{x^3}{2} + C_1 x + C_2$$

where C_1 and C_2 are constants