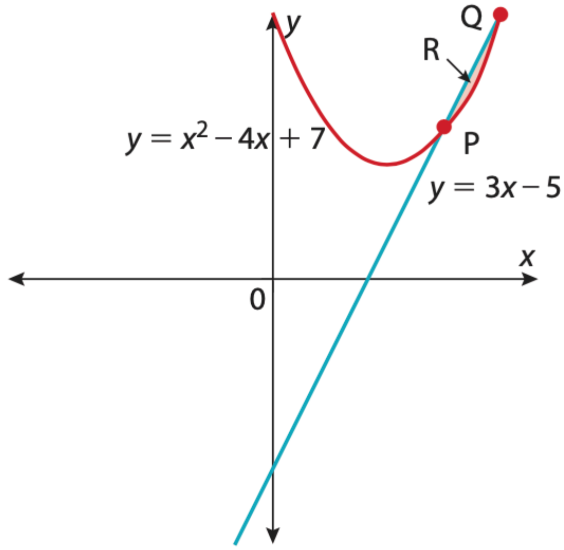
**1** The diagram shows part of the curve $y = x^2 - 4x + 7$ and part of the line $y = 3x - 5$. Find the following.

(a) The coordinates of P and Q

(b) The area of the region R




1 (a) line : $y = 3x - 5$ ...... (1)

curve : $y = x^2 - 4x + 7$ ...... (2)

At P and Q, simultaneously,

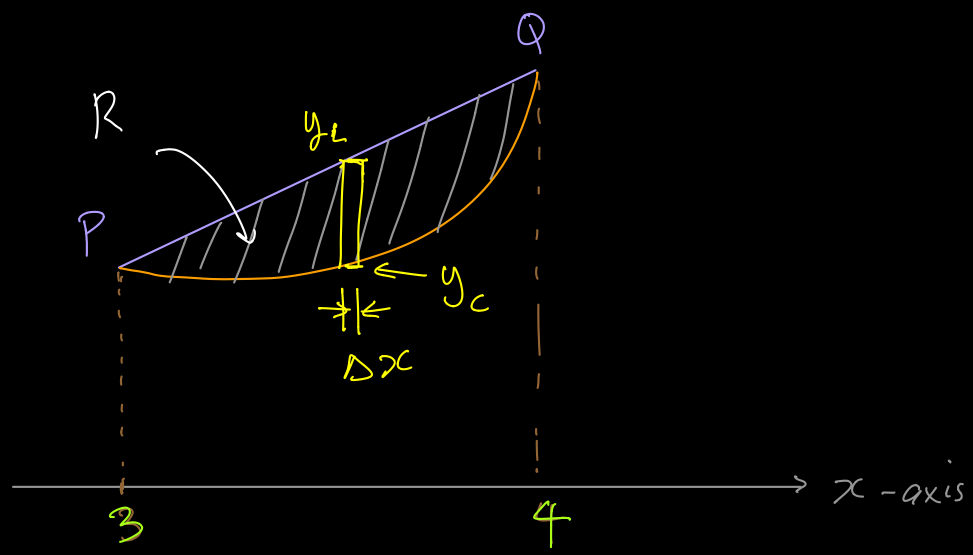
$$x^2 - 4x + 7 = 3x - 5$$
$$x^2 - 7x + 12 = 0$$
$$(x-3)(x-4) = 0$$

Either $x = 3 \Rightarrow y = 3(3) - 5 = 4$

or $x = 4 \Rightarrow y = 3(4) - 5 = 7$

ans: P(3, 4) and Q(4, 7)

1(b)




The element of area is

$$\Delta R = (y_L - y_c)\,\Delta x$$

$\therefore$ In the limit and by integration, the total area is

$$R = \int_3^4 y_L - y_c \, dx$$

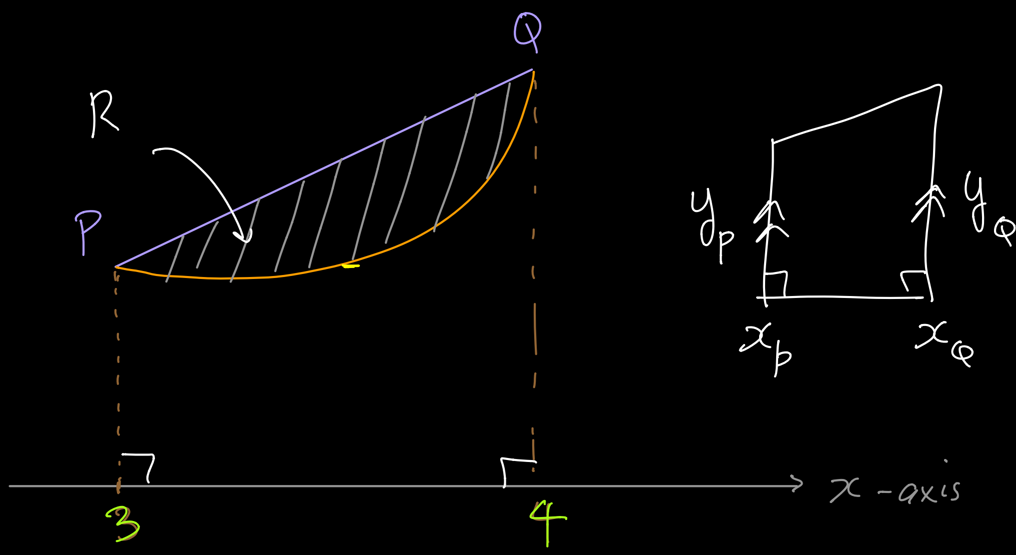
$$= \int_3^4 3x - 5 - (x^2 - 4x + 7)\, dx$$

$$= \int_3^4 -x^2 + 7x - 12 \, dx$$

$$= \left[ -\frac{x^3}{3} + \frac{7x^2}{2} - 12x \right]_3^4$$

$$= \left( -\frac{64}{3} + 56 - 48 \right) - \left( -9 + \frac{63}{2} - 36 \right)$$

$= \left( \qquad \right)$ exact



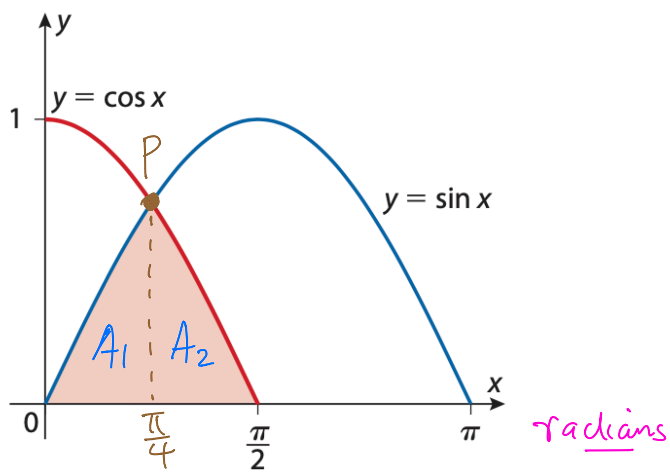

$$\text{Area } R = \left(\begin{array}{c}\text{area under}\\ \text{the line}\end{array}\right) - \left(\begin{array}{c}\text{area under}\\ \text{the curve}\end{array}\right)$$

$$= \left(\frac{y_p + y_Q}{2}\right)(x_Q - x_p) - \int_3^4 x^2 - 4x + 7 \, dx$$

$\vdots \qquad \vdots$

etc

**2** The diagram shows part of the curve $y = \sin x$ and $y = \cos x$. Calculate the area of the shaded region.




At point P,

$$\sin x = \cos x$$

$$\tan x = 1$$

$$x = \tan^{-1}(1)$$

$$x_P = \frac{\pi}{4}$$

Now, the required area is

$$A = A_1 + A_2$$

$$= 2A_1 \qquad \text{(by symmetry)}$$

$$= 2\int_0^{\frac{\pi}{4}} \sin x \, dx$$

$$= 2\left[-\cos x\right]_0^{\frac{\pi}{4}}$$

2 (cont'd.)

$$A = 2\left\{\left(-\cos\frac{\pi}{4}\right) - \left(-\cos 0\right)\right\}$$

$$= 2\left\{-\frac{\sqrt{2}}{2} + 1\right\}$$

$$= 2 - \sqrt{2} \text{ square units}$$

**4** Sketch the curve $y = (x+2)(3-x)$. Find the area of the region enclosed by the curve and the $x$-axis.

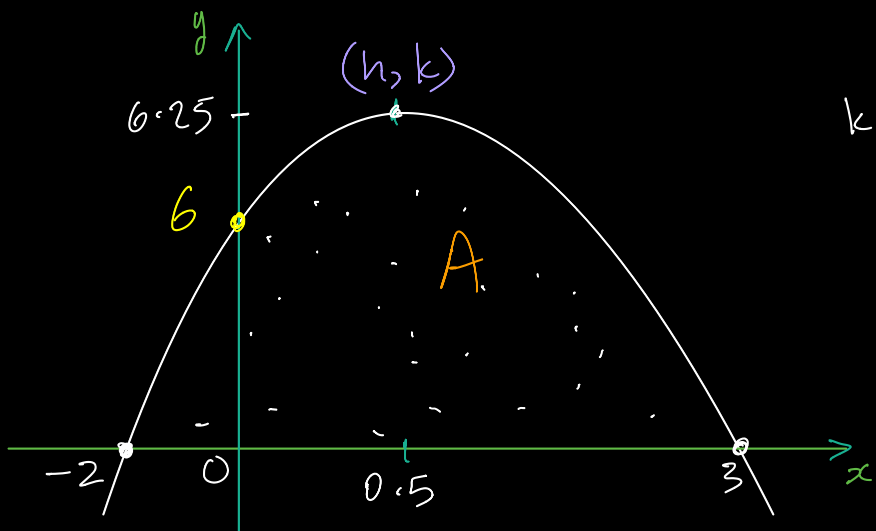
$$y = (x+2)(3-x) \quad \cdots ①$$
$$= 3x - x^2 + 6 - 2x$$
$$= -x^2 + x + 6 \quad \cdots ②$$

Roots: $x = \{-2, 3\}$

Turning Point

$$h = \frac{3+(-2)}{2}$$
$$= 0.5$$

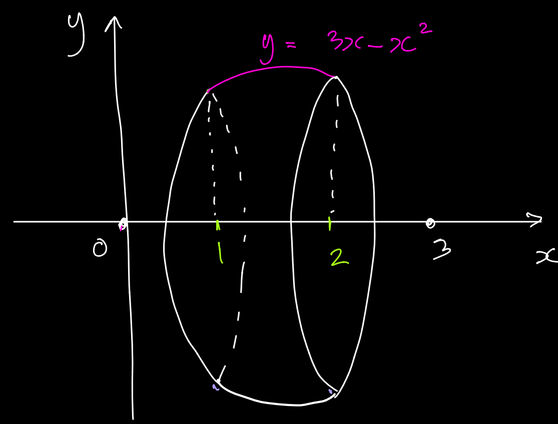
$$k = y(h)$$
$$= y(0.5)$$
$$= (2.5)(2.5)$$
$$= 6.25$$




(Not Drawn to scale)

$$\therefore \text{Area, } A = \int_{-2}^{3} y \, dx$$
$$= \int_{-2}^{3} -x^2 + x + 6 \, dx$$
$$= \left[ -\frac{x^3}{3} + \frac{x^2}{2} + 6x \right]_{-2}^{3}$$
$$= \left( -9 + \frac{9}{2} + 18 \right) - \left( +\frac{8}{3} + 2 - 12 \right)$$
$$= 10\tfrac{1}{2} - 7\tfrac{1}{3}$$
$$= 6\tfrac{1}{6} \text{ sq units}$$

**3** Find the volume of the solid formed when the area bounded by the curve $y = 3x - x^2$, the $x$-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the $x$-axis.




$$\text{Volume, } V = \int_1^2 \pi y^2 \, dx$$

$$= \int_1^2 \pi (3x - x^2)^2 \, dx$$

$$= \int_1^2 \pi (9x^2 - 6x^3 + x^4) \, dx$$

$$= \pi \left( 3x^3 - \frac{3x^4}{2} + \frac{x^5}{5} \right) \Big|_1^2$$

$$= \pi \left\{ \left( 24 - 24 + \frac{32}{5} \right) - \left( 3 - \frac{3}{2} + \frac{1}{5} \right) \right\}$$

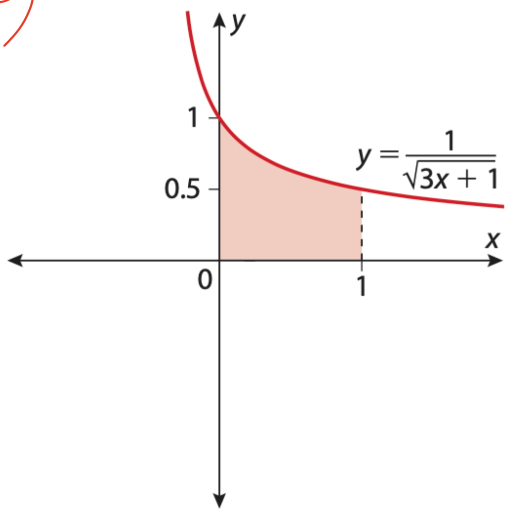
$$= \pi \left( 6\tfrac{2}{5} - 3 + 1\tfrac{1}{2} - \tfrac{1}{5} \right)$$

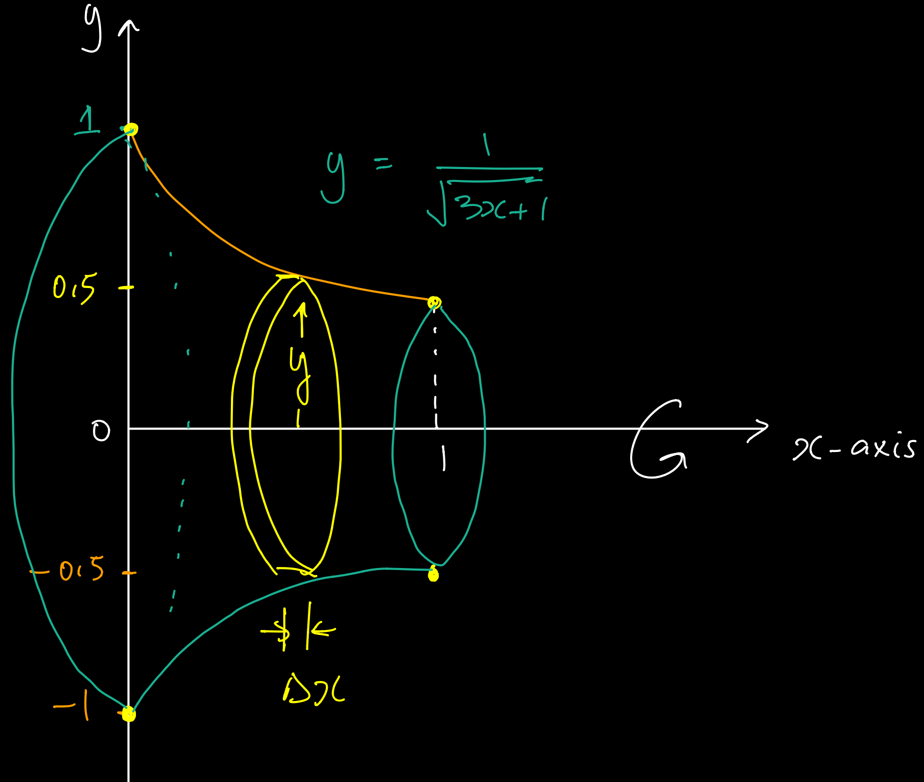
$$= \frac{47}{10}\pi$$

$$= 4.7\pi \text{ cubic units}$$

**1** The diagram shows part of the curve $= \frac{1}{\sqrt{3x+1}}$. Find the volume generated when the shaded region is rotated through 360° about the $x$-axis.
(Use $\int \frac{1}{x} = \ln|x|$.) $+C$

$dx$




The element of volume is

$$\Delta V = \pi y^2 \Delta x$$

$\therefore$ In the limit, as $\Delta x \to dx$ and $\Delta V \to dV$, using integration, Volume, $V = \int_0^1 \pi \left(\frac{1}{\sqrt{3x+1}}\right)^2 dx$

$$= \pi \int_0^1 \frac{1}{3x+1} \, dx$$

but $\int \frac{1}{ax+b} \, dx = \frac{1}{a} \ln |ax+b| + C$

$$\therefore \quad V = \pi \times \left[\frac{1}{3} \ln |3x+1|\right]_0^1$$

$$= \frac{\pi}{3} (\ln 4 - \ln 1)$$

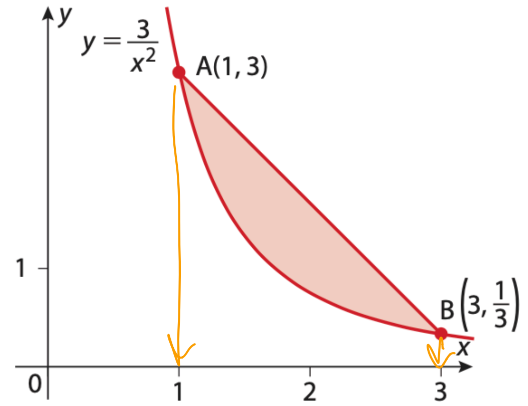
$$= \frac{\pi}{3} \ln 4 \text{ cubic units. (exact)}$$

$$\approx 1.452 \text{ cubic units (approx)}$$

**7** The points A(1, 3) and B$\left(3, \frac{1}{3}\right)$ lie on the curve $y = \frac{3}{x^2}$, as shown in the diagram.

(a) Find the equation of the line AB.

(b) Calculate the volume obtained when the shaded region is rotated through 360° about the $x$-axis.




7(a) Given A(1, 3) and B$(3, \frac{1}{3})$, then the equation of AB is

$$* \quad (y - y_A)(x_B - x_A) = (x - x_A)(y_B - y_A) \quad *$$

$$(y - 3)(3 - 1) = (x - 1)\left(\frac{1}{3} - 3\right)$$

$$2y - 6 = -\frac{8}{3}(x - 1)$$

$$2y = -\frac{8x}{3} + \frac{8}{3} + 6$$

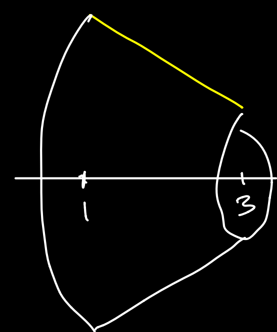
$$y = -\frac{4x}{3} + \frac{4}{3} + 3$$

$$y = -\frac{4x}{3} + \frac{13}{3}$$

trust your algebra

7(b) $V_2$ = volume due to the rotation of the line

$$= \int_1^3 \pi\left(-\frac{4x}{3}+\frac{13}{3}\right)^2 dx$$




$$= \frac{\pi}{3^2}\int_1^3 (-4x+13)^2\, dx$$

$$= \frac{\pi}{9}\left[\frac{(-4x+13)^3}{-4\times 3}\right]_1^3$$

$$= -\frac{\pi}{108}\left\{(-12+13)^3-(-4+13)^3\right\}$$

$$= -\frac{\pi}{108}\left(1^3-9^3\right)$$

$$= \frac{182\pi}{27} \text{ cubic units}$$

$V_1$ = volume due to the rotation of the curve

$$= \int_1^3 \pi\left(\frac{3}{x^2}\right)^2 dx$$

$$= \int_1^3 \pi\left(\frac{9}{x^4}\right) dx$$

$$\therefore \quad V_1 = \int_1^3 9\pi x^{-4}\,dx \qquad -4+1 = -3 \qquad \frac{9\pi x^{-3}}{-3}$$

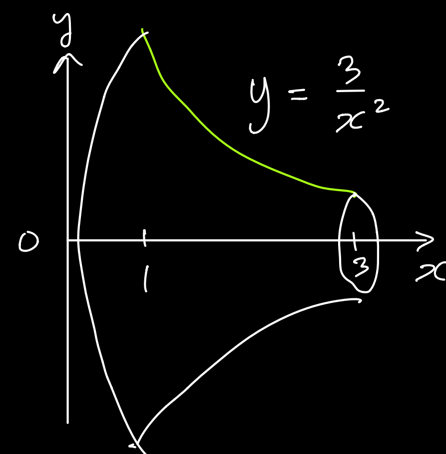
$$= -3\pi \left[ x^{-3} \right]_1^3$$

$$= -3\pi \left[ \frac{1}{x^3} \right]_1^3$$

$$= -3\pi \left( \frac{1}{27} - 1 \right)$$

$$= -\cancel{3}^{1}\pi \left( -\frac{26}{\cancel{27}_{9}} \right)$$

$$= \frac{26\pi}{9} \text{ cubic units}$$




So, the required volume is

$$\therefore V_{req} = V_2 - V_1$$

$$= \frac{182\pi}{27} - \frac{26\pi}{9}$$

$$= \frac{104\pi}{27} \text{ cubic units}$$