ADVANCED LEVEL MATHEMATICS

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LIMITS

Our First 43 Tutorial Questions

The enclosed *draft* document contains **43 problems** and basic mathematical **reviews** pertaining to your first calculus topic of Limits. These generic problem sets were compiled for our tutorial sessions.

Please take this drill seriously and feel free to alert me (as early as you can) if you have any challenges, modifications or corrections as you revise and work ahead of our regularly scheduled class sessions.

The content is as follows.

Contents

Thanks.

(Please Turn Over)

SET A EVALUATION OF LIMITS

Review I ~ Properties of Limits

Sums, Differences, Scalar Multiplications, and Products

If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist and k is a constant, then

$$\lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = \lim_{x\to a}\left[f(x) \pm g(x)\right]$$

$$\lim_{x\to a} kf(x) = k\lim_{x\to a} f(x)$$

$$\lim_{x\to a}\left[f(x)g(x)\right] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$$

Quotients

If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x) \neq 0$ exist, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

Power

If $\lim_{x\to a} f(x) = L$ and $p, L \in \mathbb{R}$, then

$$\lim_{x\to a}\left[f(x)\right]^p = \left(\lim_{x\to a} f(x)\right)^p = L^p$$

Two (2) Linear Functions

If k is a constant, then

$$\lim_{x\to a} k = k$$

$$\lim_{x\to a} x = a$$

Polynomials and Rational Functions

Provided that $q(a) \neq 0$, then

$$\lim_{x\to a} \frac{p(x)}{q(x)} = \frac{p(a)}{q(a)}$$

Review II ~ Some <u>Axiomatic</u> Properties of Zero, 0 and Infinity, $\pm\infty$

Addition and Subtraction

For any real number a, then

$$a+0 \equiv 0+a \equiv a$$

$$a+(-0) \equiv (-0)+a \equiv a$$

Multiplication

For any real number a such that $a>0$, then

$$a \bullet 0 \equiv 0 \bullet a \equiv 0$$

$$a \bullet \infty \equiv \infty \bullet a \equiv \infty$$

$$(-a) \bullet \infty \equiv \infty \bullet (-a) \equiv -\infty$$

Division (Please note well!)

For any real number a such that $a>0$, then

$\frac{a}{0} = \infty$ is more precisely stated as $\lim_{h \to 0^-}\left(\frac{a}{h}\right) = \lim_{h \to 0^+}\left(\frac{a}{h}\right) \to +\infty$

$\frac{-a}{0} = -\infty$ i.e., $\lim_{h \to 0^-}\left(\frac{-a}{h}\right) = \lim_{h \to 0^+}\left(\frac{-a}{h}\right) \to -\infty$

$\frac{0}{0}$ is **indeterminate**[1] and is therefore left undefined

$\frac{\infty}{\infty}$ is indeterminate

$\frac{\infty}{0}$ is **undefined**[2]

$\frac{0}{\infty}$ is undefined

Check the web[3] for more precise mathematical definitions.

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[1] Indeterminate means “not able to be stated or described in an exact way”

[2] Undefined in mathematics means “the operation cannot be interpreted in any meaningful way”

[3] http://en.wikipedia.org/wiki/Extended_real_number_line#Arithmetic_operations

Problems involving the evaluation of limits (for polynomials)

Find the value of the following **limits**, if they exist.

1. $\lim_{x\to 2}\left(3x^2-5x+2\right)$

2. $\lim_{x\to -1}\left(x^3-2x^2+x-3\right)$

3. $\lim_{x\to 3}(x-1)^2(x+1)$

4. $\lim_{x\to -1}\left(x^2+1\right)(1-2x)^2$

5. $\lim_{x\to 0}\left(1-5x^3\right)$

6. $\lim_{x\to 5}\left(\frac{x+3}{x-5}\right)$

(Perform polynomial division to express $\frac{x+3}{x-5}$ in the form $A+\frac{B}{x-5}$. Then apply the limits. Comment on the need for this step. How could this approach be of benefit for another problem type?)

7. $\lim_{x\to 3}\left(\frac{2x+3}{x-3}\right)$ (Use the experience of Problem 6)

8. $\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)$

9. $\lim_{x\to 3}\left(\frac{9-x^2}{x-3}\right)$

10. $\lim_{x\to 5}\left(\frac{x^2-3x-10}{x-5}\right)$

11. $\lim_{x\to 2}\left(\frac{x^2+x-6}{x-2}\right)$

12. $\lim_{x\to 4}\frac{(x+1)(x-4)}{(x-1)(x-4)}$

13. $\lim_{x\to 0}\frac{x\left(x^2-1\right)}{x^2}$.

14. $\lim_{x\to\infty}\left(\frac{x+2}{x-7}\right)$

15. $\lim_{x\to -2}\left(\frac{x^2-x-6}{x^2+3x+2}\right)$

16. $\lim_{x\to 1}\left(\frac{x^2+4x-5}{x^2-1}\right)$

17. $\lim_{x\to 4}\left(\frac{\sqrt{x}-2}{x-4}\right)$ (hint: $x-4=\left(\sqrt{x}\right)^2-2^2$, which is the "difference of 2 squares")

18. $\lim_{x\to 9}\left(\frac{\sqrt{x}-3}{x-9}\right)$

19. $\lim_{x\to 1}\left(\frac{x-1}{\sqrt{x}-1}\right)$

20. $\lim_{x\to 2}\left(\frac{6x+10}{2x-4}\right)$

<u>Important notes for the reader</u>

1. Repeat problems 1 to 20 by changing all the limits to "$x\to\infty$". This will be a very useful exercise.

2. We will return to problems involving trigonometric functions in the next session.

(Please Turn Over)

SET B CONTINUITY OF FUNCTIONS

Determine if the following functions are continuous at the specified value of *x*.

21. $f(x) = 5x^2 - 6x + 1 \; ; \qquad x = 2$

22. $f(x) = \dfrac{x+2}{x+1} \; ; \qquad x = 1$

23. $f(x) = \dfrac{x+1}{x-1} \; ; \qquad x = 1$

24. $f(x) = \dfrac{\sqrt{x}-2}{x-4} \; ; \qquad x = 4$

25. $f(x) = \dfrac{2x-4}{3x-2} \; ; \qquad x = 2$

26. $f(x) = \dfrac{\sqrt{x}-2}{x-4} \; ; \qquad x = 2$

27. $f(x) = \begin{cases} x+1 & \text{if } x \le 2 \\ 2 & \text{if } x > 2 \end{cases} \; ; \qquad x = 2$

28. $f(x) = \begin{cases} 0 & \text{if } x > 1 \\ x-1 & \text{if } x \le 1 \end{cases} \; ; \qquad x = 1$

29. $g(x) = \begin{cases} x+1 & \text{if } x \le 0 \\ x-1 & \text{if } x > 0 \end{cases} \; ; \qquad x = 0$

30. $g(x) = \begin{cases} x^2+1 & \text{if } x \le 3 \\ 2x+4 & \text{if } x > 3 \end{cases} \; ; \qquad x = 3$

31. $h(x) = \begin{cases} \dfrac{x^2-1}{x+1} & \text{if } x < -1 \\ x^2-3 & \text{if } x \ge -1 \end{cases} \; ; \qquad x = -1.$

(Please Turn Over)

Find the value(s) of *x* for the following functions are *discontinuous*.

32. $f(x) = 3x^2 - 6x + 9$

33. $f(x) = x^5 - x^3$

34. $f(x) = \dfrac{3x+3}{x+1}$

35. $f(x) = \dfrac{x^2-1}{x+1}$

36. $\dfrac{x}{x^2 - x}$

37. $\dfrac{x}{(x+5)(x-1)}$

38. $\dfrac{x^2-2x+1}{x^2-x-2}$

39. $\dfrac{3x-2}{(x+2)(x-6)}$

40. $h(x) = \tan x$

Additional problems (conditions are continuous and discontinuous functions)

41. Find the values of the constants *A* and *B* such that a function *f*(*x*) is continuous for all values of *x* where $f(x) = \begin{cases} Ax^2 + 5x - 9 & \text{if } x < 1 \\ B & \text{if } x = 1 \\ (3-x)(A-2x) & \text{if } x > 1 \end{cases}$

42. Find the value of the constant *A* such that a function *f*(*x*) is continuous for all values of *x* where $f(x) = \begin{cases} Ax - 3 & \text{if } x < 2 \\ 2 - x + 3x^2 & \text{if } x \geq 2 \end{cases}$

43. Find the value of the constant *B* such that a function *f*(*x*) is discontinuous for a value of *x* where $f(x) = \begin{cases} 1 + 3x & \text{if } x < 4 \\ Bx^2 + 2x - 3 & \text{if } x \geq 4 \end{cases}$. What is the *x* value at the point of discontinuity?

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